Vibration Analyst Category IV Equations

FORCES

Mass Unbalance

$$F = Me\left(\frac{2\pi N}{60}\right)^2$$

M = W/g
W = weight of rotor or balance weight, lb
e = rotor eccentricity or radius of balance weight, in
g = gravitational constant, 386.1 in/s^2
N = RPM

Spring Force

F = Kx

K = stiffness of spring, lb/in
x = relative deflection, in

Damping Force

$F = C\dot{x}$

C = damping constant, lb-s/in
$\dot{x}$ = relative velocity

Inertia Force

$F = M\ddot{x}$

M = mass, lb-s^2/in
$\ddot{x}$ = acceleration, in/s^2

MOTIONS

Velocity (in/s)

$V = D(2\pi f)$

D = peak displacement, in
f = frequency, cycles/s (CPS)
$\pi = 3.14$

Acceleration

$A = V(2\pi f)$

A = acceleration, in/s^2
1 g = 386.1 in/s^2

FREQUENCIES

Bearing Frequencies

$$FTF = \left(\frac{\Omega}{2}\right)\left[1 - \left(\frac{B}{P}\right)\cos CA\right]$$

$$BPFI = \left(\frac{N}{2}\right)\Omega\left[1 + \left(\frac{B}{P}\right)\cos CA\right]$$

$$BPFO = \frac{N}{2}\Omega\left[1 - \left(\frac{B}{P}\right)\cos CA\right]$$

$$BSF = \left(\frac{P}{2B}\right)\Omega\left[1 - \left(\frac{B}{P}\right)^2 \cos^2 CA\right]$$

FTF = fundamental train frequency
BPFI = ball pass frequency, inner race
BPFO = ball pass frequency, outer race
BSF = ball spin frequency

CA = contact angle
Ω = machine speed
N = number of rolling elements
P = pitch diameter, in
B = ball or roller diameter, in

Natural Frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

k = stiffness, lb/in
m = w/g
w = weight, lb
g = gravitational constant, 386.1 in/s^2
f_n = natural frequency of a single-degree-of-freedom system, Hz

Roll Frequency

$$f = \frac{V}{5\pi D}$$

V = web velocity, ft/min
D = roll diameter, in
f = frequency, Hz

SIGNAL PROCESSING

Dynamic Range

$$dB = 20 \log \frac{V_m}{V_r}$$

$$\frac{V_m}{V_r} = 10^{\frac{dB}{20}}$$

V_m = voltage measured
V_r = voltage reference
dB = decibels

RMS

peak = 1.414 rms

Resolution

Resolution = (frequency span x window noise factor x 2)/#FFT lines

window noise factor =
1.0 for uniform window
1.5 for Hanning window
3.8 for flat top window

Data Acquisition Time (DAT)

DAT = # FFT lines/frequency span

VIBRATION RESPONSE

Rotor Vibration Response

$$r = \frac{e\left(\frac{f}{f_n}\right)^2}{\sqrt{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\zeta\frac{f}{f_n}\right]^2}}$$

r = rotor vibration response, in peak
e = eccentricity of mass, in
f = forcing frequency, Hz
f_n = natural frequency, Hz
ζ = ratio of damping to critical damping

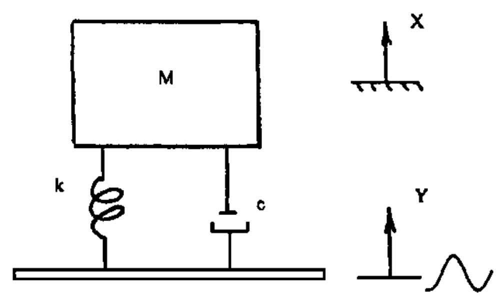
Phase

$$\tan\phi = \frac{2\zeta\frac{f}{f_n}}{1-\left(\frac{f}{f_n}\right)^2}$$

ϕ = phase, deg

Force Transmission/Base Motion

$$\frac{X}{Y} = \frac{F_{TR}}{F_0} = \frac{\sqrt{1+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$



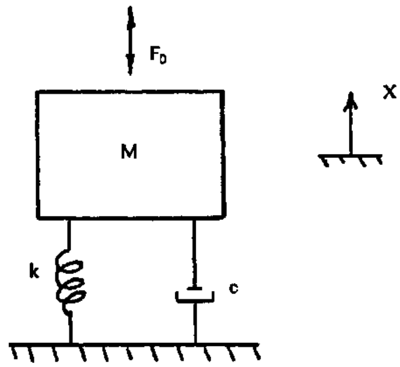
Base Motion

F_{TR} = transmitted force, lb
F_0 = exciting force, lb
ω = forcing frequency, rad/s
ω_n = natural frequency, rad/s
ζ = damping ratio
k = spring stiffness, lb/in
X = mass motion, inches peak
Y = base motion, inches peak

Forced Vibration Response

$$X = \frac{\frac{F_0}{k}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

F_{TR} = transmitted force, lb
F_0 = exciting force, lb
ω = forcing frequency, rad/s
ω_n = natural frequency, rad/s
ζ = damping ratio
k = spring stiffness, lb/in
X = mass motion, inches peak
Y = base motion, inches peak



Forced Vibration Response

STABILITY THRESHOLD

$$\omega_S = \omega_0\sqrt{\frac{c}{g}}$$

ω_s = dimensionless stability threshold
ω_0 = journal operating speed, rad/s
c = bearing radial clearance, in
g = gravitational constant

AMPLIFICATION FACTOR

Half Power

$$AF = \frac{N_{CR}}{N_2 - N_1}$$

AF = amplification factor
N_{CR} = critical speed, RPM
N_2, N_1 = half power points, RPM

Phase Change

$$AF = \frac{\pi N_{CR}}{360} \frac{\Delta\phi}{\Delta N}$$

AF = amplification factor
N_{CR} = critical speed, RPM
$\Delta\phi$ = phase change, degrees
ΔN = speed change, RPM

DAMPING RATIO – TIME DOMAIN

$$\delta = \text{log decrement} = \frac{1}{n} \ell n \frac{x_0}{x_n}$$

$$\zeta = \frac{c}{c_c} \cong \frac{\delta}{2\pi} \text{ damping ratio}$$

n = number of cycles

$$\frac{x_0}{x_n} = \text{amplitude ratio in n cycles}$$

$$\frac{c}{c_c} = \frac{1}{2AF} = \text{damping ratio}$$

where C = damping constant, lb s/in

C_C = 2 mω_n critical damping, $\frac{\text{lb} - \text{sec}}{\text{in.}}$

AF = amplification factor

M = system mass, $\frac{\text{lb sec}^2}{\text{in.}}$

ω_n = system natural frequency, $\frac{\text{rad}}{\text{sec}}$

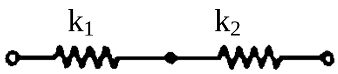
DUAL CONSTANT – DUAL CHANNEL

Damping Calculation $$Q = \frac{(f_a / f_b)^2 + 1}{(f_a / f_b)^2 - 1}$$

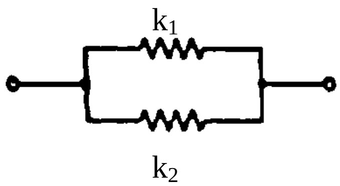
where f_a = frequency above resonance, where the real or imaginary part of the transfer function reaches a peak

f_b = frequency below resonance, where the real or imaginary part of the transfer function reaches a peak of opposite sign

TABLE OF SPRING STIFFNESS



$$k = \frac{1}{1/k_1 + 1/k_2}$$



$$k = k_1 + k_2$$

$$k = \frac{EI}{\ell}$$

I = moment of inertia of circular cross-sectional area

$$I = \frac{\pi D^4}{64}$$

ℓ = total length

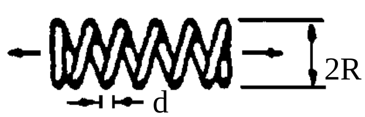
$$k = \frac{EA}{\ell}$$

A = cross-sectional area

$$k = \frac{GJ}{\ell}$$

J = torsion constant of circular cross section

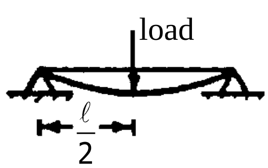
$$J = \frac{\pi D^4}{32}$$



$$k = \frac{Gd^4}{64nR^3}$$

n = number of turns

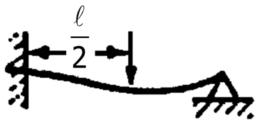
$$k = \frac{3EI}{\ell^3}$$



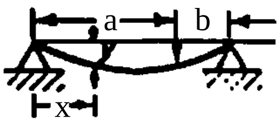
$$k = \frac{48EI}{\ell^3}$$

$\frac{\ell}{2}$

$$k = \frac{192EI}{\ell^3}$$

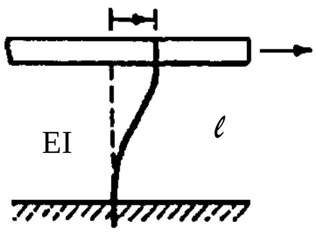


$$k = \frac{768EI}{7\ell^3}$$

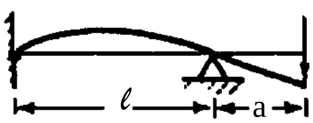


$$k = \frac{3EI\ell}{a^2b^2}$$

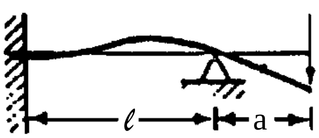
$$\ell = a + b$$



$$k = \frac{12EI}{\ell^3}$$



$$k = \frac{3EI}{(\ell + a)a^2}$$



$$k = \frac{24EI}{a^2(6\ell + 8a)}$$